 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **STATISTICS**

FIRST SEMESTER – NOVEMBER 2012

# ST 1820/1815 - ADVANCED DISTRIBUTION THEORY

Date : 01/11/2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

SECTION - A

Answer ALL questions. Each carries TWO marks: (10 x 2 = 20 marks)

1. If X is the number of heads obtained when a coin is tossed twice, then show that X is a

random variable.

2. Define distribution function of a random variable and state its properties.

3. Let the distribution function of x be

F(x) =

Show that the distribution of X is neither discrete nor continuous.

4. Write the pdf of truncated binomial, left truncated at ‘0’ and find its mgf.

5. Prove that geometric distribution satisfies lack of memory property.

6. Let X1, X2, … , Xn be iid non-negative and integer valued random variables. If

X(1) = Min{ X1, X2, … , Xn } is a geometric random variable, then show that X1 is

geometric.

7. Find the pgf of power-series distribution and hence obtain its mgf.

8. Let (X1, X2) ~ BB(n, p1, p2, p12). Find the marginal distributions of X1 and X2.

9. State and prove additive property of bivariate Poisson distribution.

10. Define compound distribution, when (i) is discrete, (ii) is continuous.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks: (5 x 8 = 40 marks)

11. Let X be a random variable with distribution function

F(x) =

Find (i) the decomposition of F,

(ii) mgf of F.

12. Find the mean, variance and the mgf of truncated Poisson distribution, left truncated at ‘0’.

13. State and prove a characterization of Poisson distribution through pdf.

14. Let X1 and X2 be iid Poisson random variables with parameter . Then show that the

conditional distribution of X1 | X1 + X2 = n is B(n, ).

15. Verify that binomial, Poisson and log-series distributions are power-series distributions.

16. State and prove Skitovitch theorem for normal distributions.

17. Let X1, X2, X3 be independent normal variables such that E(X1) = 1, E(X2) = 3,

E(X3) = 2 and V (X1) = 2, V(X2) = 2 and V(X3) = 3. Examine the independence of the

following pairs:

(i) X1 + X2 and X1 – X2

(ii) X1 + X2 -2X3 and X1 – X2

(iii) 2X1 + X3 and X2 – X3 .

18. Obtain the mgf of inverse Gaussian distribution.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks: ( 2 x 20 = 40 marks)

19(a) Let X have a power-series distribution. Find the first cumulant k1 and derive the

recurrence formula for finding rth cumulant kr . Hence obtain kr for Poisson

distribution. (10)

(b) If (X1, X2) ~ BB(n, p1, p2, p12), then show that X1 | X2 = x2  U1 + V1 , where

U1 ~ B (n – x2 , ), V1 ~ B(x2 ,) and U1  is independent of V1. (10)

20(a) For log-normal distribution show that mean > median > mode. (10)

(b) Let X1 ~ G(, p1) , X2 ~ G(, p2) and X1 ╨ X2. Then show that

(i) X1 + X2 ~ G(, p1 + p2 ),

(ii) X1 / (X1 + X2 ) ~ Beta distribution of first kind,

(iii) (X1 + X2 ) ╨ (X1 / (X1 + X2)). (10)

21(a) Let X ~ IG(. Then show that( 2 ) /  2X) ~  2(1). (8)

(b) Let (X1, X2) ~ BVN(µ1, µ2, ,, ). Find the conditional distribution of

(i) X2 | X1 = x1 and (ii) X1 | X2 = x2. (12)

22(a) Let (X1, X2) ~ BVN(µ1, µ2, ,, ). Derive the mgf of (X1, X2) at (t1, t2). (10)

(b) Define non-central t – distribution and derive its pdf. (10)

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